ALPHAOMEGAMATH
YOUR MATHEMATICS SPECIALIST

ADDITIONAL MATHEMATICS CHEAT SHEET

SME500 SINGAPORE AWARD WINNER - 2022 -

Trigonometric Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$

$\sec\theta = \frac{1}{\cos\theta}$ $\quad \operatorname{cosec}\theta = \frac{1}{\sin\theta}$

$\sin(-\theta) = -\sin\theta$ $\quad \sin(90-\theta) = \cos\theta$

$\cos(-\theta) = \cos\theta$ $\quad \cos(90-\theta) = \sin\theta$

$\tan(-\theta) = -\tan\theta$ $\quad \tan(90-\theta) = \cot\theta$

$\sin(90+\theta) = \cos\theta$ $\quad \sin(180-\theta) = \sin\theta$

$\cos(90+\theta) = -\sin\theta$ $\quad \cos(180-\theta) = -\cos\theta$

$\tan(90+\theta) = -\cot\theta$ $\quad \tan(180-\theta) = -\tan\theta$

$\sin(180+\theta) = -\sin\theta$ $\quad \sin^2\theta + \cos^2\theta = 1$

$\cos(180+\theta) = -\cos\theta$ $\quad 1+\tan^2\theta = \sec^2\theta$

$\tan(180+\theta) = \tan\theta$ $\quad 1+\cot^2\theta = \operatorname{cosec}^2\theta$

Addition and subtraction Formulae, Double Angle Formulae & R-formulae

$\sin 2A = 2\sin A\cos A$ $\quad a\sin\theta \pm b\cos\theta = R\sin(\theta \pm \alpha)$

$\cos 2A = \cos^2 A - \sin^2 A$ $\quad a\sin\theta \pm b\cos\theta = R\sin(\theta \pm \alpha)$

$= 2\cos^2 A - 1$ $\quad R = \sqrt{a^2+b^2}$

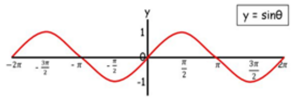
$= 1 - 2\sin^2 A$ $\quad \alpha = \tan^{-1}(\frac{b}{a})$

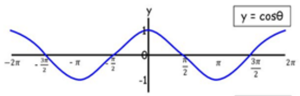
$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$ $\quad \sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

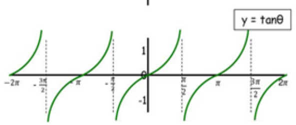
$\cos\frac{A}{2} = \sqrt{\frac{1+\cos A}{2}}$ $\quad \cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\sin\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}}$ $\quad \tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

x	0°	30°	45°	60°	90°	180°	270°	360°
sin x	0	1/2	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos x	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	½	0	-1	0	1
tan x	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	∞	0





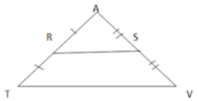


y = a sin bx + c, y = a cos bx + c, y = a tan bx + c

max: a + c, min = - a + c

period: 360/b (for sin and cos), 180/b (for tan)

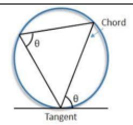
shift: C



Midpoint theorem

R and S are midpoints of AT and AV

RS is parallel to TV and RS=1/2 (TV)



Alternate Segment Theorem

Differentiation (Notation: f ′(x) = dy/dx = d[ ]/dx)

dy/dx = gradient function. dy/dx=0 for stationary / turning points.

☑ Rules of differentiation

$\frac{d[k]}{dx} = 0, k \text{ is a const.}$

$power\ rule: y = kx^n, \frac{dy}{dx} = knx^{n-1}$

$chain\ rule: \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$Product\ rule: \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$Quotient\ rule: \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

☑ Rules of trigo differentiation

$\frac{d(\sin x)}{dx} = \cos x$ $\quad \frac{d[\tan(ax+b)]}{dx} = a\sec^2(ax+b)$

$\frac{d(\cos x)}{dx} = -\sin x$ $\quad \frac{d(\sin^n x)}{dx} = n\sin^{n-1}x\cos x$

$\frac{d(\tan x)}{dx} = \sec^2 x$ $\quad \frac{d(\cos^n x)}{dx} = -n\cos^{n-1}x\sin x$

$\frac{d[\sin(ax+b)]}{dx} = a\cos(ax+b)$ $\quad \frac{d(\tan^n x)}{dx} = n\tan^{n-1}x\sec^2 x$

$\frac{d[\cos(ax+b)]}{dx} = -a\sin(ax+b)$

Differentiation of logarithm and exponential functions.

$\frac{d(\ln x)}{dx} = \frac{1}{x}$ $\quad \frac{d(e^x)}{dx} = e^x$ $\quad \frac{d}{dx}(e^{f(x)}) = f'(x)e^{f(x)}$

$\frac{d[\ln(f(x))]}{dx} = \frac{f'(x)}{f(x)}$ $\quad \frac{d(e^{ax+b})}{dx} = ae^{ax+b}$

To test for stationary points use **1st** derivative test:

Sub x⁻ (value smaller than x). If dy/dx<0	Sub x, if dy/dx=0	Sub x⁺ (value slightly bigger than x) if dy/dx>0
/	—	\
Maximum point		
\	—	/
Minimum point		
/	—	/
\	—	\
Point of inflexion		

Or **2nd** derivative test: (does not test for point of inflexion, back to 1st test)

$\frac{d^2y}{dx^2} > 0\,(\min)\ ;\ \frac{d^2y}{dx^2} < 0\,(\max)$ Hint: Use 2nd derivative test if the questions highlight the use of maximum or minimum. If not use 1st derivative test.

Integration

$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$ $\quad \int \frac{1}{(ax+b)} = \frac{1}{a}\ln(ax+b) + c$

☑Integration of trigonometric functions and exponential functions

$\int \sin x\,dx = -\cos x + c$ $\quad \int \sin ax\,dx = -\frac{\cos ax}{a} + c$ $\quad \int e^x\,dx = e^x + c$

$\int \cos x\,dx = \sin x + c$ $\quad \int \cos ax\,dx = \frac{\sin ax}{a} + c$ $\quad \int e^{ax+b}dx = \frac{1}{a}e^{ax+b} + c$

$\int \tan x\,dx = -\ln\cos x + c$ $\quad \int \sec^2 ax\,dx = \frac{\tan ax}{a} + c$

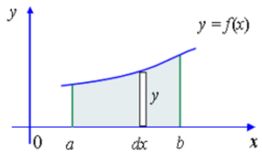
$\int \sec^2 x\,dx = \tan x + c$ $\quad \int \sin(ax+b)\,dx = -\frac{\cos(ax+b)}{a} + c$

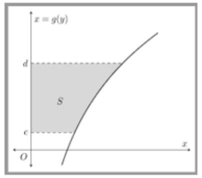
$\int \cos(ax+b)\,dx = \frac{\sin(ax+b)}{a} + c$ $\quad \int \tan ax\,dx = \frac{-\ln\cos ax}{a} + c$

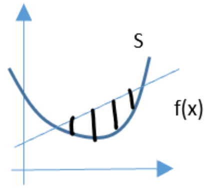
$\int \sec^2(ax+b)\,dx = \frac{\tan(ax+b)}{a} + c$

☑Area under the curve

$\int_a^b y\,dx$ $\quad \int_c^d x\,dy$







To find area bounded by a line and curve. Use the higher graph minus the lower graph. Ie $\int_a^b f(x) - g(x)\,dx$

If graph falls below x axis or left of y axis, take positive value.

☑Kinematics

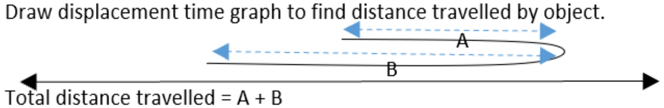
Differentiate (v= ds/dt) Differentiate (a = dv/dt)

Displacement ⟷ Velocity ⟷ Acceleration

$s = \int v\,dt$ Integrate $v = \int a\,dt$ Integrate

Useful notes: v=0m/s, object turns. Maximum velocity/acceleration do differentiation. 1st 4 seconds = 0 to 4s, 4th second = 3s to 4s

Draw displacement time graph to find distance travelled by object.

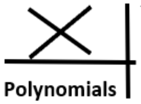


Total distance travelled = A + B

ADDITIONAL MATHEMATICS CHEAT SHEET

Simultaneous Equations

☑use of **substitution** method.
☑simplify log/indices/surds/ fractional/coordinate geometry/ word problems or other forms.
☑use of quadratic factorisation.



-2 sets of answers.
x_1 = ____ , y_1 = ____
x_2 = ____ , y_2 = ____

Polynomials

☑ f(x) = D(x) x Q(x) + R(x)
Dividend = divisor x quotient + remainder

☑Factor theorem. If $x \pm a$, sub $f(\mp a)$=0.

☑Reminder theorem. If $x \pm a$, sub $f(\mp a)$ = remainder.

☑Methods involved: comparing coefficients, substitution, simultaneous equations

☑Special algebraic identities

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

☑Cubic equations/ factorization (1)Use calculator to find first factor. (2)Do long division method. (3)Cross factorization ☑Partial fractions

Type 1: $\dfrac{px+q}{(ax+b)(cx+d)} = \dfrac{A}{ax+b} + \dfrac{B}{cx+d}$

Type 2: $\dfrac{px+q}{(ax+b)^2} = \dfrac{A}{ax+b} + \dfrac{B}{(ax+b)^2}$

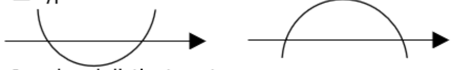
Type 3: $\dfrac{px+q}{(ax+b)(x^2+c^2)} = \dfrac{A}{ax+b} + \dfrac{Bx+C}{x^2+c^2}$

Type 4: Improper fraction> check degree > do long division

Quadratic Equations and inequalities

D=b^2-4ac where ax^2+bx+c=0.

☑Type 1:

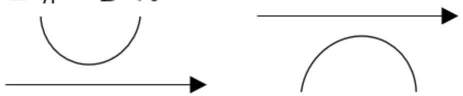


2 real and distinct roots.
(intersects/cuts at 2 points and has real and distinct roots)

☑Type 2: $D = 0$

1 real and repeated root.
(touch/intersects the curve at 1 point and tangent to the curve)

☑Type 3: $D < 0$



No real roots. (does not interest the curve, curve or expression is always positive/negative)

☑Type 4: $D \geq 0$ (hybrid of either type 1/2) Real roots. (intersects/touch the curve)

*To show for type 1,3 or 4, use completing the square.

☑Quadratic Inequalities
(x-a)(x-b) < 0 , a<x<b ; b>a
(x-a)(x-b) > 0 , x<a or x>b

Binomial theorem

$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$

$T_{r+1} = \binom{n}{r}a^{n-r}b^r$; if T_5 → r = 4.

☑ For independent term , use general term and find r for x^0.

$\binom{n}{2} = \dfrac{n(n-1)}{2!}$; $\binom{n}{3} = \dfrac{n(n-1)(n-2)}{3!}$

Coordinate Geometry

Midpoint theorem: $\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$. Parallel lines have equal gradient.

Equation of straight line: $y = mx + c$ or $y_1 - y_2 = m(x_1 - x_2)$. For perpendicular lines: $m_1 \times m_2 = -1$.

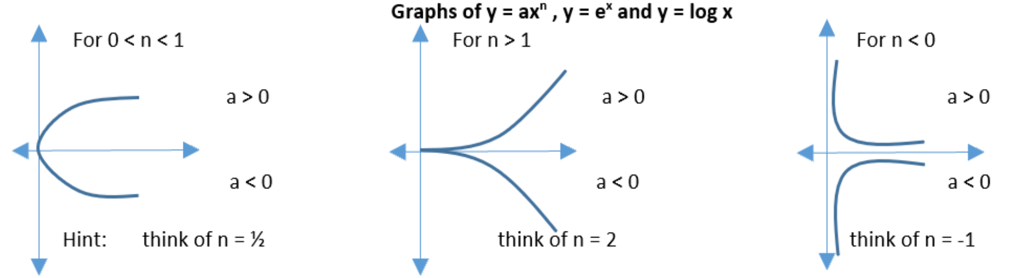
Area= $\dfrac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix} = \dfrac{1}{2}[(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)]$

Length of a line = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

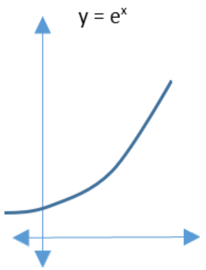
Equation of Circles (1) $(x-a)^2 + (y-b)^2 = r^2$: Centre of circle (a,b) and radius, r.

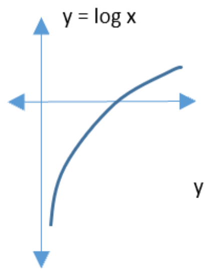
(2) $x^2 + y^2 + 2gx + 2fy + c = 0$: Centre of circle (-g,-f), radius = $\sqrt{g^2 + f^2 - c}$

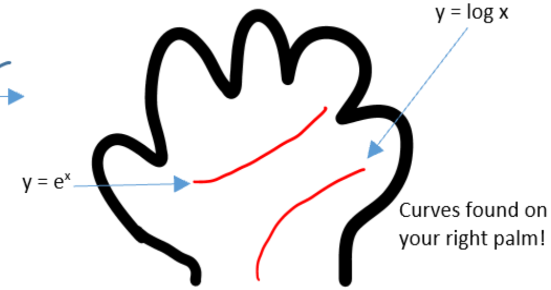
Graphs of y = ax^n , y = e^x and y = log x



n is a rational number







Rules for Indices, Surds and Logarithms

Law 1: $a^n \times a^m = a^{n+m}$ Law 2: $a^n \div a^m = a^{n-m}$ Law 3: $(a^n)^m = a^{nm}$ Law 4: $a^n \times b^n = (ab)^n$ Law 5: $a^n \div b^n = \left(\dfrac{a}{b}\right)^n$ Law 6: $a^0 = 1$ Law 7: $a^{-n} = \dfrac{1}{a^n}$ Law 8: $a^{\frac{1}{n}} = \sqrt[n]{a}$ Law 9: $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

$a^x = b \Leftrightarrow \log_a b = x$

$a\sqrt{x} + b\sqrt{x} = (a+b)\sqrt{x}$

$\dfrac{\sqrt{x}}{\sqrt{y}} = \dfrac{\sqrt{x}}{\sqrt{y}} \times \dfrac{\sqrt{y}}{\sqrt{y}} = \dfrac{\sqrt{xy}}{y}$

$\dfrac{1}{\sqrt{x}+\sqrt{y}} = \dfrac{1}{\sqrt{x}+\sqrt{y}} \times \dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}} = \dfrac{\sqrt{x}-\sqrt{y}}{x-y}$

$\log_a xy = \log_a x + \log_a y$

$\log_a x^n = n\log_a x$

$\log_a a = 1$

$a\sqrt{x} - b\sqrt{x} = (a-b)\sqrt{x}$

$\log_a \dfrac{x}{y} = \log_a x - \log_a y$

$\log_a 1 = 0$

$\log_a x = \dfrac{\log_b x}{\log_b a}$

$\sqrt{x}\sqrt{y} = \sqrt{xy}$